Geometry 3-3 Study Guide Proving Lines Parallel (pp 162-165)

Attendance Problems. Write the converse of each statement.

1. If $a = b$, then $a + c = b + c$.

2. If $m\measuredangle A + m\measuredangle B = 90°$, then $\measuredangle A$ & $\measuredangle B$ are complementary.

3. If $AB + BC = AC$, then A, B, and C are collinear.

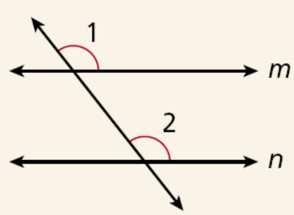
I can use the angles formed by a transversal to prove two lines are parallel.

<u>Common Core</u>

- **CC.9-12.G.CO.9** Prove geometric theorems about lines and angles.
- **CC.9-12.G.CO.12** Make formal geometric constructions with a variety of tools and methods.

Recall that the converse of a theorem is found by exchanging the hypothesis and conclusion. The converse of a theorem is not automatically true. If it is true, it must be stated as a postulate or proved as a separate theorem.

Postulate 3-3-1 **Converse of the Corresponding Angles Postulate**

THEOREM	HYPOTHESIS	CONCLUSION
If two coplanar lines are cut by a transversal so that a pair of corresponding angles are congruent, then the two lines are parallel.	$\angle 1 \cong \angle 2$ 	$m \parallel n$

Q: Why didn't the two parallel lines recognize each other?
A: Because they had never met

"The mark of an immature man is that he wants to die nobly for a cause, while the mark of a mature man is that he wants to live humbly for one." -- *William Stekel*

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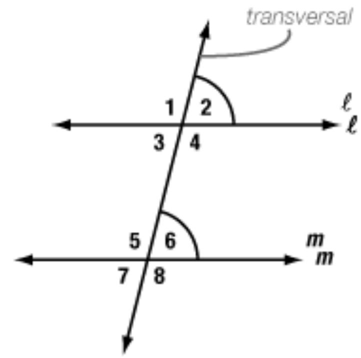
<u>Video Example 1</u>.

A. Given: <2 congruent <6

Prove: l || m

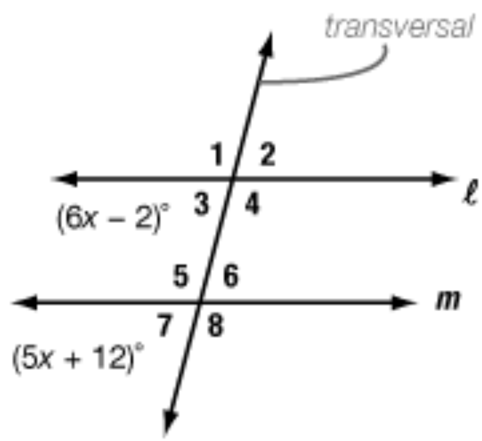


B. Given: m<3 = (6x - 2)^o

m<7=(5x + 12)^o

x = 14

Prove: l || m



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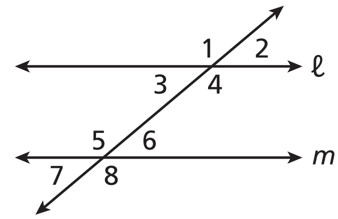
1 Using the Converse of the Corresponding Angles Postulate

Use the Converse of the Corresponding Angles Postulate and the given information to show that $\ell \parallel m$.



A $\angle 1 \cong \angle 5$

$\angle 1 \cong \angle 5$	*∠1 and ∠5 are corresponding angles.*
$\ell \parallel m$	*Conv. of Corr. ∠s Post.*

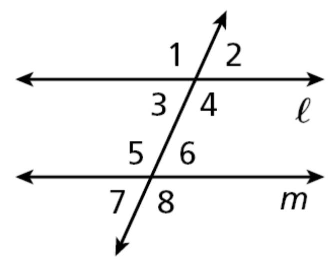
B $m\angle 4 = (2x + 10)°, m\angle 8 = (3x - 55)°, x = 65$

$m\angle 4 = 2(65) + 10 = 140$	*Substitute 65 for x.*
$m\angle 8 = 3(65) - 55 = 140$	*Substitute 65 for x.*
$m\angle 4 = m\angle 8$	*Trans. Prop. of Equality*
$\angle 4 \cong \angle 8$	*Def. of ≅ ∠s*
$\ell \parallel m$	*Conv. of Corr. ∠s Post.*

<u>Example 1</u>.

A. Given: <4 is congruent to <8

Prove: l || m



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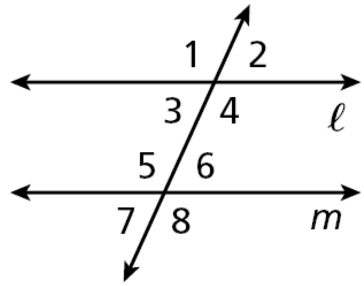
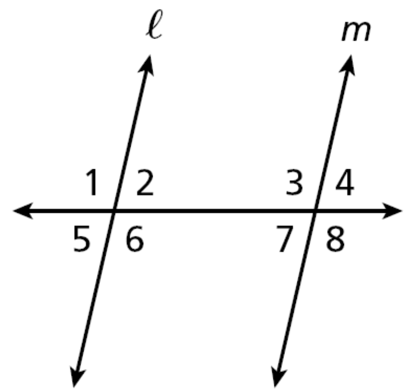
B. Given: m<3 = (4x - 80)º
m<7 = (3x - 50)º
x = 30
Prove: l || m



<u>Guided Practice</u>**. Use the Converse of the Corresponding Angles Postulate and the given information to show that *ℓ* || *m*.**

4. $m\measuredangle 1 = m\measuredangle 3$



5. $m\measuredangle 7 = (4x + 25)°$
$m\measuredangle 5 = (5x + 12)°, x = 13$

Postulate 3-3-2 **Parallel Postulate**

Through a point *P* not on line ℓ, there is exactly one line parallel to ℓ.

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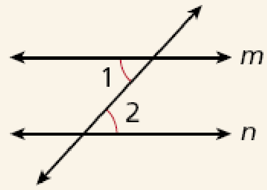
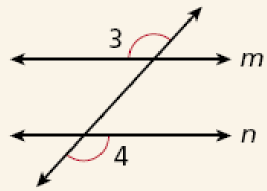
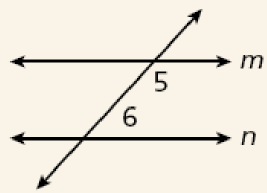
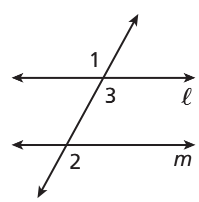
Theorems: Proving Lines Parallel

THEOREM		HYPOTHESIS	CONCLUSION
3-3-3	**Converse of the Alternate Interior Angles Theorem** If two coplanar lines are cut by a transversal so that a pair of alternate interior angles are congruent, then the two lines are parallel.	$\angle 1 \cong \angle 2$ 	$m \parallel n$
3-3-4	**Converse of the Alternate Exterior Angles Theorem** If two coplanar lines are cut by a transversal so that a pair of alternate exterior angles are congruent, then the two lines are parallel.	$\angle 3 \cong \angle 4$ 	$m \parallel n$
3-3-5	**Converse of the Same-Side Interior Angles Theorem** If two coplanar lines are cut by a transversal so that a pair of same-side interior angles are supplementary, then the two lines are parallel.	$m\angle 5 + m\angle 6 = 180°$ 	$m \parallel n$

PROOF **Converse of the Alternate Exterior Angles Theorem**



Given: $\angle 1 \cong \angle 2$
Prove: $\ell \parallel m$
Proof: It is given that $\angle 1 \cong \angle 2$. Vertical angles are congruent, so $\angle 1 \cong \angle 3$. By the Transitive Property of Congruence, $\angle 2 \cong \angle 3$. So $\ell \parallel m$ by the Converse of the Corresponding Angles Postulate.

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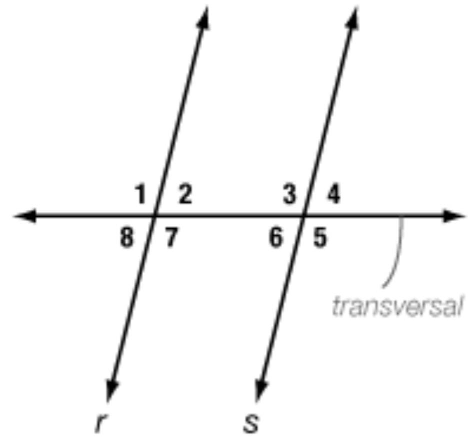
<u>Video Example 2</u>. Refer to the diagram. Use the given information and the theorems you have learned to show that $r \parallel s$.

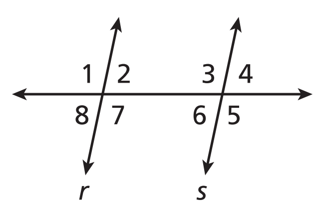


4. $m\measuredangle 4 = m\measuredangle 8$

5. m<3 = (4x + 2)º, m<2 = (8x - 2)º, and x = 15.

2 Determining Whether Lines are Parallel

Use the given information and the theorems you have learned to show that $r \parallel s$.



A $\angle 2 \cong \angle 6$

$\angle 2 \cong \angle 6$ — *$\angle 2$ and $\angle 6$ are alternate interior angles.*

$r \parallel s$ — *Conv. of Alt. Int. ∠s Thm.*

B $\mathbf{m\angle 6 = (6x + 18)^\circ, m\angle 7 = (9x + 12)^\circ, x = 10}$

$m\angle 6 = 6x + 18$

$= 6(10) + 18 = 78^\circ$ — *Substitute 10 for x.*

$m\angle 7 = 9x + 12$

$= 9(10) + 12 = 102^\circ$ — *Substitute 10 for x.*

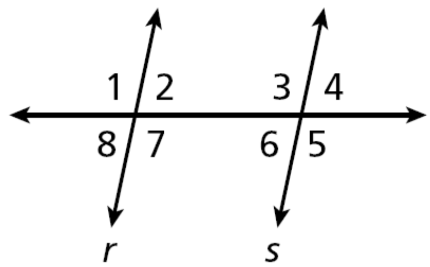
$m\angle 6 + m\angle 7 = 78^\circ + 102^\circ$

$= 180^\circ$ — *$\angle 6$ and $\angle 7$ are same-side interior angles.*

$r \parallel s$ — *Conv. of Same-Side Int. ∠s Thm.*

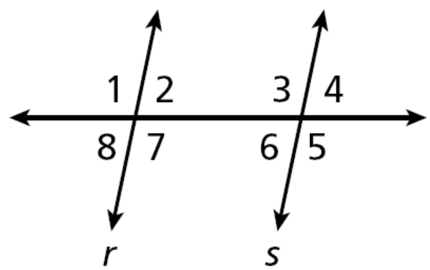
Geometry 3-3 Study Guide Proving Lines Parallel (pp 162-165)

Example 2. Refer to the diagram. Use the given information and the theorems you have learned to show that *r* || *s*.

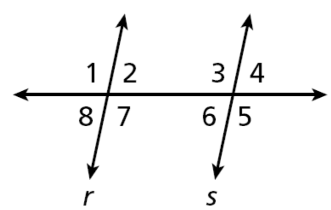


A. $m\measuredangle 4 = m\measuredangle 8$

B. m<2 = $(10x + 8)°$, m<3 = $(25x – 3)°$, $x = 5$



Guided Practice. Refer to the diagram. Use the given information and the theorems you have learned to show that *r* || *s*.

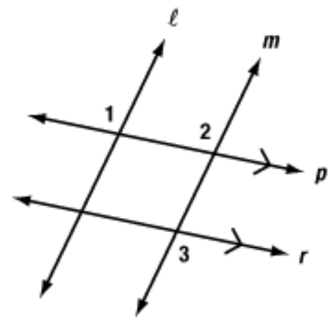


6. $m\measuredangle 4 = m\measuredangle 8$

7. $m\measuredangle 3 = 2x°$
$m\measuredangle 7 = (x + 50)°$
$x = 50.$

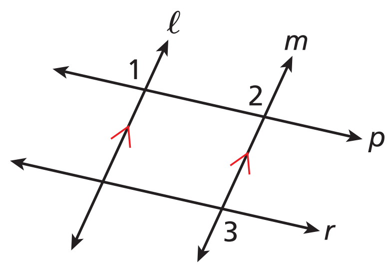
Geometry 3-3 Study Guide Proving Lines Parallel (pp 162-165)

Video Example 3.
Given: r || p, <1 is congruent to <3.
Prove: l || m



3 Proving Lines Parallel

Given: $\ell \parallel m$, $\angle 1 \cong \angle 3$
Prove: $r \parallel p$

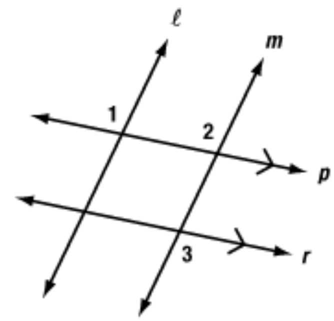


Proof:

Statements	Reasons
1. $\ell \parallel m$	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** Corr. ∡s Post.
3. $\angle 1 \cong \angle 3$	**3.** Given
4. $\angle 2 \cong \angle 3$	**4.** Trans. Prop. of ≅
5. $r \parallel p$	**5.** Conv. of Alt. Ext. ∡s Thm.

Geometry 3-3 Study Guide Proving Lines Parallel (pp 162-165)

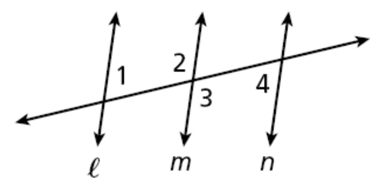
<u>Example 3</u>.
Given: r || p, <1 is congruent to <3.
Prove: l || m



8. <u>Guided Practice</u>.

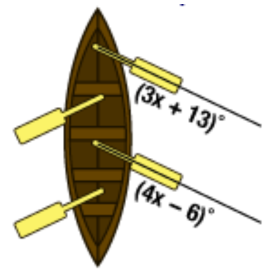
Given: $\measuredangle 1 \cong \measuredangle 4$
$\measuredangle 3$ & $\measuredangle 4$ are supplementary.

Prove: $\ell \parallel m$



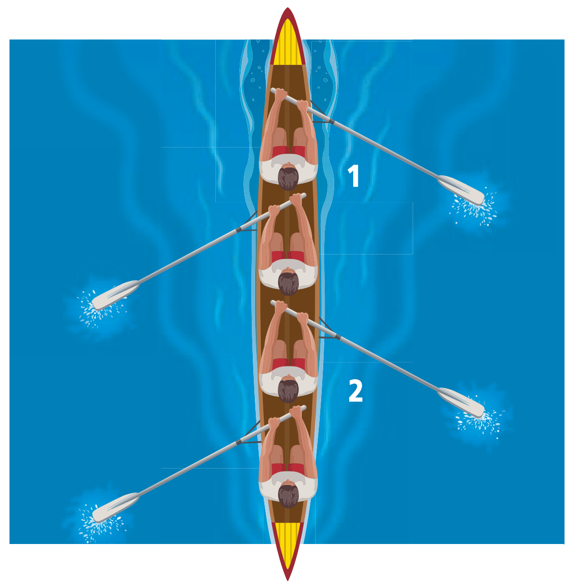
<u>3-3 Proving Lines Parallel</u> *(pp 166)* 13, 15, 19, 21, 22.

Video Example 4. During a race, all members of a rowing team should keep the oars parallel on each side. If m<1 = (3x + 13)º, m<2 = (4x - 6)º, and x = 19, show that the oars are parallel.

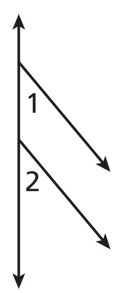


4 Sports Application

During a race, all members of a rowing team should keep the oars parallel on each side. If $m\angle 1 = (3x + 13)^\circ$, $m\angle 2 = (5x - 5)^\circ$, and $x = 9$, show that the oars are parallel.



A line through the center of the boat forms a transversal to the two oars on each side of the boat.



$\angle 1$ and $\angle 2$ are corresponding angles. If $\angle 1 \cong \angle 2$, then the oars are parallel.

Substitute 9 for x in each expression:

$m\angle 1 = 3x + 13$

$= 3(9) + 13 = 40^\circ$ *Substitute 9 for x in each expression.*

$m\angle 2 = 5x - 5$

$= 5(9) - 5 = 40^\circ$ *$m\angle 1 = m\angle 2$, so $\angle 1 \cong \angle 2$.*

The corresponding angles are congruent, so the oars are parallel by the Converse of the Corresponding Angles Postulate.

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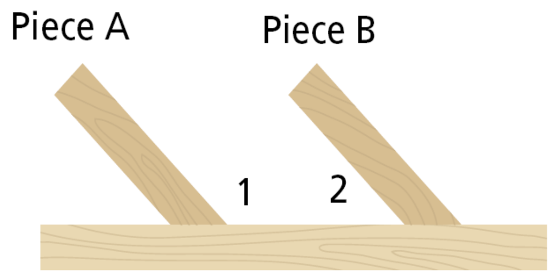
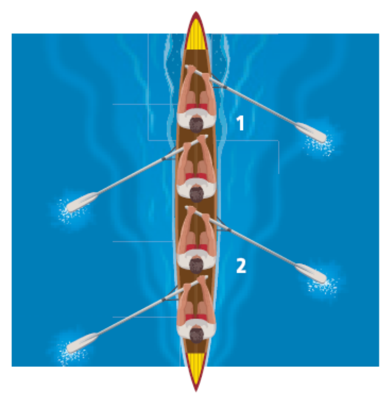
Example 4.

A carpenter is creating a woodwork pattern and wants two long pieces to be parallel. m<1= (8*x* + 20)° and m<2 = (2*x* + 10)°. If *x* = 15, show that pieces A and B are parallel.



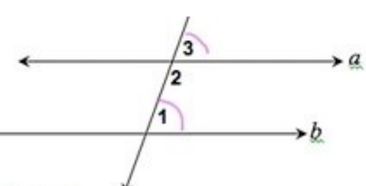
9. Guided Practice. Suppose the corresponding angles on the opposite side of the boat measure (4*y* – 2)° and (3*y* + 6)°, where *y* = 8. Show that the oars are parallel.



3-3 Proving Lines Parallel *(pp 166)* 13, 15, 19, 21, 22, 37-39, 41-42.

Proof of the Converse of the Same-Side Interior Angles Theorem
Given: ∠1 and ∠2 are supplementary
Prove: *a* || *b*
(5-6 steps)

≈ def.



Statements	Reasons
1) ∠1 ; ∠2 are supple.	Given
2) ∠1 + ∠2 = 180°	def. of supple.
3) ∠2 + ∠3 = 180°	Angle add. post.
4) ∠1 + ∠2 = ∠2 + ∠3	Substitution
5) ∠1 ≅ ∠3	sub
6) a // b	corr. ∠ post. converse